2019-20 Example Scenario

Farm Insurance

A program of The Actuarial Foundation

Modeling The Future Challenge

Introduction

Providing healthy, nutritious food for our continuously growing population is critical to our prosperity as a nation and global society. However, environmental stressors and a growing population put ever increasing pressures on our farms to produce the crops and livestock required at affordable prices.

Climate change is expected to significantly alter important environmental factors that could have large effects on agricultural sectors across the country. Changing temperatures, droughts, floods, and other severe weather events will cause major upheavals to many industries across the country, but perhaps none more so than the agricultural industry.

In this scenario, you are given information from three farms in Manitoba, Canada: Abbington Farm, Barton Farm, and Calistoga Farm. Each farm owner has tracked their profits (in thousands of dollars) and the weather over the past 20 years. The farmers organized their annual weather reports into three general categories, noted as *typical, dry, or stormy.*

Typical weather happens most years and had no major weather events causing adverse effects to the farmer's crops. Typical weather generally yields the best results for the farmers. In unusually dry years, farmers saw some crop die-off due to droughts and other related environmental stressors, leading to smaller profits. Occasionally, in what is noted as a "stormy" year, one or more severe storms wiped out a portion of the farmers' crops for that year, severely damaging their production and profits.

The data-set provided includes columns for the year, the farm, the weather category, and the profit reported by the three farmers. The questions on the following pages require you to analyze this data and some additional information provided in the questions to help understand the future profit and risks for these farmers.

Level 1 Questions:
Basic Statistics & Probability

The first thing you are tasked with is to conduct some basic mathematical analysis of the reports the farmers gave. This will help us understand what has happened in the past 20 years.

1. **Using the data from the past 20 years, calculate the probabilities of each type of weather that the farmers reported.**

   Typical weather = 14/20 = 70%

   Dry weather = 4/20 = 20%

   Severe storm weather = 2/20 = 10%

2. **Compare the average profits for each of the three farms across all years and all weather categories. Which farm had the highest average profit?**

   Barton Farm has highest average profit of $74.65 thousand.

   Abbington Farm has average profit $68.05 thousand.

   Calistoga Farm has average profit $65.45 thousand.

3. **Compare the standard deviations of the profits for each of the three farms across all years and all weather categories. Which farm had the greatest variability?**

   Abbington Farm has st. dev. $20.19 or $20.85 thousand using sample STDEV

   Barton has greatest standard deviation of $22.98 thousand (or $24.11 thousand if sample standard deviation calculated)

   Calistoga has standard deviation $21.76 or $22.21 thousand

4. **Is the farm with the overall highest average profit that you identified in question #2, the farm with the highest average profit for each type of weather? Identify which farm had the highest profits for each of the three weather types.**

Farm	Avg Dry	Avg Storm	Avg Typical	Avg Total
Abbington	$50K	$20.5K	$80K	$68.05K
Barton	$47.75K	$28K	$89K	$74.65K
Calistoga	$44.75	$16K	$78.43K	$65.45K

   No - in Dry weather, Abbington Farm has highest average profit of $50k. Barton has highest average profit in stormy years ($28K) and in typical years ($89k).

5. **Construct a 95% confidence interval for the mean profit for Barton Farm during typical weather years. What are the upper and lower bounds on the farm's profit for typical weather?**

   Confidence interval for Barton Farm typical = 89 +/- (1.96) * 5.6975 / SQRT(16)
   = (85.96, 92.04)

Level 2 Questions: Projecting Trends

6. **Calculate the probability of exactly two storms in the next ten years.**

   Probability of exactly 2 storms in next 10 years = $\binom{10}{2}(.1)^2(.9)^8 = 0.194$

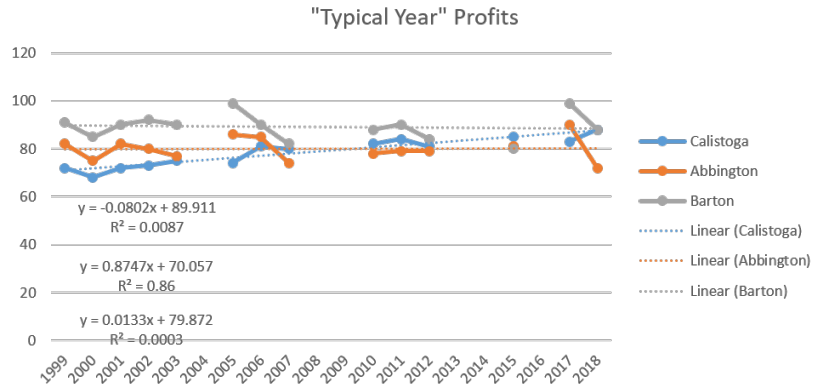
7. **Calculate the probability of at least two storms in the next ten years.**

   Probability of at least 2 storms in next 10 years = 0.264

8. **Calculate the probability that the second storm from now will happen in the tenth year from now.**

   Probability of 2nd storm in 10th year = (9)(.1)(.9)^8 (.1) = .039

9. **How quickly is each farm is growing during "typical" weather years? Calculate the least-squares linear regression for each of the three farms' typical weather annual profits. Create a graph plotting the profits and the trend line for each of the three farms. Which farm is growing the fastest in typical weather years?**



   Barton and Abbington farm are not growing fast. The least squares line for Barton is slightly negative, meaning it is actually shrinking in profit. Calistoga is the only farm that seems to be growing significantly.

10. **All three farms indicated there were two stormy years over the past two decades (2004, and 2013). Which farm was affected by these storms the most? Using the expected values from your linear regression for typical weather, what was the average profit lost for each farm due to stormy weather?**

Farm	2004 predicted	2013 predicted	2004 Actual	2013 Actual
Abbington	79.95	80.07	22	19
Barton	89.43	88.71	29	27
Calistoga	75.31	83.18	17	15

   Abbington's average loss to stormy years was 59.51, Barton's was 61.07, and Calistoga's was 63.24. So Calistoga is most affected by storms.

11. Use least-squares linear regression on typical weather years only to predict the profit for Calistoga Farm in 2020 (assuming that 2020 also has typical weather).

For Calistoga, the linear regression equation is: y = 0.8747*(Year)+70.057, where the year 1999 = 1 (so the year 2020, is the number 22 in this equation). So plugging this in, we get an expected profit in 2020 of: 89.3, or $89,300 in dollars.

12. If the year 2020 has a 10% chance of being stormy, a 20% chance of dry weather, and a 70% chance of being typical, what is the expected value of each farm's profit for the year 2020? Use the average loss due to stormy and dry weather from the past 20 years to calculate the expected value of each farm's profit for 2020.

Farm	2020 Typical Profit ($)	2020 Stormy Profit ($)	2020 Dry Profit ($)
Abbington	80.16	20.65	50.11
Barton	88.15	27.08	47.09
Calistoga	89.3	26.06	51.97

Here we need to calculate the expected value of the profit for each farm given the likelihood of each type of weather happening in the year 2020. We know the probabilities for each type of weather, so we need to calculate what the profit would be for each. We do this using the linear regression equations from previous questions, and calculating the average loss in profit for Dry and Stormy weather from the previous years (two years of storms: 2004, 2013; and 4 years of dry: 2008, 2009, 2014, 2016). The expected values for the profit in 2020 for each farm given each type of weather are noted in the table above. To get the expected value of the profit for the farm, we multiply these by their probabilities of happening: 0.1 for Stormy weather, 0.2 for Dry weather and 0.7 for typical weather.

Abbington Expected Profit = (0.7) * (80.16) + (0.1) * 20.65 + (0.2) * 50.11 = $68.20K

Barton Expected Profit = (0.7) * (88.15) + (0.1)*27.08 + (0.2) * 47.09 = $73.83K

Calistoga Expected Profit = (0.7) * (89.3) + (0.1)*26.06 + (0.2) * 51.97 = $75.51K

Level 3 Questions: Risks & Insurance

In order to alleviate the risk from severe weather, Abbington Farm is considering an insurance company that offers an annual policy for a $3,000 premium that would pay $20,000 in case of a severe storm.

13. Construct a probability distribution for the expected profit using the average profit for the 20 years of historical data if Abbington Farm buys this insurance policy.

Using the average profit for Abbington in each type of weather, we can see that Abbington's profit probability table with the insurance would be:

profit	probability
77	0.7
47	0.2
37.5	0.1

14. Calculate the expected value of Abbington's profit and the standard deviation of the expected profit with insurance.

Expected Profit = (0.7) * 77 + (0.2) * 47 + (0.1) * 37.5 = **$ 67.05K**

Standard deviation =sqrt((77-67)2 *(0.7) + (47-67.05)2 * (0.2) + (37-67)2 * (0.1))= **$15.49 K**

15. For Abbington Farm, compare your answers to #2 and #14 and interpret the values in context.

Abbington Farm has average profit without insurance is $68.05 thousand. With insurance the average profit goes down a little to $67.05 thousand. This makes sense because the insurance company has to make some profit from selling their policy to Abbington.

16. For Abbington Farm, compare your answers to #3 and #15 and interpret the values in context.

Abbington Farm has st. dev. $20.19 without insurance. With insurance the standard deviation drops to $15.49K. This makes sense because the insurance policy is removing some variability from their potential profit loss in stormy years.

17. What is the expected profit for the insurance company on its policy for Abbington Farms?

The Insurance Company's profit is: (0.7) * 3,000 + (0.2) * 3,000 + (0.1) * -17,000 = $1000.

18. Give a mathematical reason why the insurance policy could be considered a good idea by the farmer.

Insurance could be good for the farmer because it reduces the standard deviation / variability of the profit to protect against very low profits.

19. Give a mathematical reason why the insurance policy might NOT be considered a good idea for the farmer.

Insurance might look bad for the farmer because the expected profit is lower when buying insurance.

Level 4 Questions: Critical thinking recommendations

20. What factors should the farmer consider in making the decision of whether to purchase the insurance policy noted in the previous questions?

The farmer should consider the likelihood of having a stormy year. They should also consider if there are other things they can do to help mitigate the risk posed by having a stormy year (i.e. better drainage systems or different crops that can handle more severe weather). They should also consider the potential loss if they do not have the insurance. Could their farm survive a big loss, or would it put them out of business? Is the farmer okay with the risk of that loss or would they be more comfortable knowing that they have coverage if there is a big loss?

The owner of Abbington Farm decides to consult a climate scientist to analyze what they think the climate will be like in the coming years. The climate scientist tells Abbington's owner that the probability of a stormy year now is actually 0.18%, not the 10% they've seen over the past 20 years. Likewise, the probability of typical weather and dry weather are both 4% less than what the farmer has seen in the past 20 years (66% for Typical and 16% for dry weather).

21. With this new information, would you recommend that Abbington Farm buy the insurance? Why or why not? Explain with mathematics.

With the new probabilities, Abbington's expected profit without insurance is:

Expected Profit = (0.66) * $80 + (0.16) * $50 + (0.18) * $20.5 = $64.49K

Expected Profit with Insurance = (0.66) * $77 + (0.16) * $47 + (0.18) * $37.5 = $65.09K

So Abbington's expected profit with insurance is now higher than without, so they should buy the insurance.

22. How much would these changes in climate probabilities affect the insurance company's expected profit on selling a policy to Abbington Farm?

The insurance company's profit is now: (0.66) * $3K + (0.16) * $3K + (0.18) * -$17K = -$6K

23. If you are consulting for the insurance company, how much would you recommend they increase their policy price to maintain the same expected profit as before?

$1K = (0.66) * X + (0.16) * X + (0.18) * (X - 20). So X = **$4.6K** for their new premium to equal the same profit as before ($1K).

24. If the insurance company was considering adding a deductible to the policy it is offering to Abbington Farms instead of increasing the premium, how much should the deductible be to keep the company's expected profit the same as it was? Explain with mathematics.

$1K = (0.66) * $3K + (0.16) * $3K + (0.18) * (3 – (20 – D)). So 0.18D = $1.6 thousand. **Then Deductible = $8,889.**

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